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CHANNEL CONFIGURATION - K-STATION MODEL

I. INTRODUCTION

This is a companion note to [6] in which the channel configuration for the packet radio network is investigated. In [6] we used a one station model which seems appropriate for a suburban area. In this note we use a K-station model (which more closely represents an urban area); and address the same problem. Namely, given a channel capacity C, the question is whether one should split the channel such that one part is used for transmission from stations to terminals and the second part for transmission from terminals to stations; or alternatively, should one use the whole capacity for transmission in both directions. The criteria for making the decision is based on maximum throughput and average delay.

It is assumed that the terminals originate the traffic and the stations respond with a packets for every packet that they correctly receive. We consider the ideal case (see assumptions) in which there is complete coordination among the stations so that if more than one station receive a packet, then the response will come from a single station. The slotted ALOHA mode of channel access is used.

The results of [3] are used to obtain a closed form solution of the maximum throughput for the common channel and split channel configurations as a function of K and α. The conclusion of [6] which suggests the use of a common channel configuration is further enhanced when K (>1) stations are used. Another observation suggests to use the smallest possible number of stations and locate them so as to minimize the possible interference among packets transmitted from them.

Notation

σ - rate of packet origination in packets per slot

γ - rate of packet origination plus packet retransmission in packets per slot

D - expected packet delay; from the time that the packet is ready for transmission by the originating device until the time that the last bit has been successfully received by the receiving device.

k - number of slots over which retransmission is uniformly distributed after a constant period of waiting

α - ratio of packet origination rate from stations to terminals to packet origination rate from terminals to stations

- subscript 1 will denote quantities from terminals to stations, and subscript 2 quantities from stations to terminals

i-s-K - denotes the configuration; i=1,2 for the common and split channel configurations, respectively; s indicates the slotted ALOHA mode of channel access; K is the number of stations

Assumptions

1. The point process of packet origination and the point process of packet origination plus retransmission are both assumed Poisson.

2. The probability that a packet is blocked given the packet is new is assumed equal to the probability that the packet is blocked given it has been blocked before; both are assumed to be independent of k (see [4] and [5]).

3. In evaluating the delays, we ignore the propagation time assuming that it is small compared to the transmission time.

4 Packet transmission time T=1; thus, σ is also the utilization or throughput per slot; and γ is also the channel traffic per slot. Note that in the equations we assume T=1 for the 2-channel as well as 1-channel configurations, for convenience. In drawing the delay curves, however, it is taken into account that the 2-channel configuration is a split of the 1-channel. Thus, assuming the same packet size, the transmission time in the 2-channel configuration is T=2. The number of slots delay is therefore normalized to the 1-channel configuration.

5. Although a ratio a is used, it is assumed that the process of packet origination from stations to terminals is independent of the process or packet origination from terminals to stations.

6. A no-capture system is assumed. That is, whenever two or more packets overlap, none of them is correctly received.

7. The stations and terminals are assumed to operate in an urban area, in the sense that every device is within an effective transmission distance to any other device.

8. The stations share equally the load.

9. It is assumed that there is coordination among the stations so that the response to an originating packet will be transmitted from a single station. This assumes hierarchical routing as described in [2], or alternatively that there are point-to-point channels between stations for obtaining this coordination.

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II. MAXIMUM UTILIZATION

Configuration 1-s-K

The equations relating the throughput and channel traffic from terminals to stations and from stations to terminals are:

$$\sigma_1 = \gamma_1(1-\gamma_2)^K e^{-\gamma_1} \tag{1}$$

$$\sigma_2 = K\gamma_2(1-\gamma_2)^{K-1} e^{-\gamma_1} \tag{2}$$

where γ_2 is the rate from one station and σ_2 is the total throughput from all stations, per slot. To find the maximum throughput of of this configuration we have to maximize $(\sigma_1+\sigma_2)$ subject to:

$$\alpha = \frac{\sigma_2}{\sigma_1} = \frac{K\gamma_2}{\gamma_1(1-\gamma_2)} \tag{3}$$

$$\gamma_2 \leq 1 \tag{4}$$

It was obtained in [3] that the condition for maximum throughput for the slotted ALOHA case is $\Sigma\gamma_i = 1$. Thus, in this case:

$$K\gamma_2 + \gamma_1 = 1 \tag{5}$$

We now solve (3) and (5) for the channel traffics at the maximum throughput γ_{1M} and γ_{2M}:

$$\gamma_{2M} = \frac{K + \alpha(K+1) - \sqrt{[K + \alpha(K+1)]^2 - 4\alpha^2 K}}{2K\alpha} \tag{6}$$

$$\gamma_{1M} = 1 - K\gamma_{2M} \tag{7}$$

Note that by taking the negative sign of the root we also satisfy $\gamma_{2M} \leq 1$ and $\gamma_{1M} \geq 0$. The positive root results in $\gamma_{2M} \geq 1$ and $\gamma_{1M} \leq 0$. The maximum throughput is thus given by:

$$\sigma_M^{1-s-K} = (1+\frac{1}{\alpha})\ K\ \gamma_{2M}\ (1-\gamma_{2M})^{K-1}\ e^{-\gamma_{1M}} \qquad (8)$$

Configuration 2-s-K

The throughput equations for this case are:

$$\sigma_1 = \gamma_1\ e^{-\gamma_1} \qquad (9)$$

$$\sigma_2 = K\gamma_2\ (1-\gamma_2)^{K-1} \quad , \quad \gamma_2 \leq 1 \qquad (10)$$

Again we have to maximize $(\sigma_1+\sigma_2)$ subject to $\sigma_2/\sigma_1 = \alpha$. The condition $\Sigma\gamma_{iM} = 1$ results $\gamma_{1M} = 1$, $\gamma_{2M} = 1/K$, thus $\sigma_{1M} = 1/e$, $\sigma_{2M} = (1-1/K)^{K-1}$. The solution for the maximum of the sum is given by (we divide by 2 to account for having a split channel):

$$\sigma_M^{2-s-K} = \begin{cases} \frac{1}{2}\ (1+\alpha)\ \frac{1}{e} & , \quad \alpha \leq e\ (1-\frac{1}{K})^{K-1} \\ \frac{1}{2}\ (1+\frac{1}{\alpha})\ (1-\frac{1}{K})^{K-1} & , \quad \alpha > e\ (1-\frac{1}{K})^{K-1} \end{cases} \qquad (11)$$

In (11) the channel from terminals to stations is "saturated" for small values of α, and the channel from stations to terminals for large α.

In the general case for the split channel one has:

$$\sigma_1 = \sigma_1(\gamma_1)$$

$$\sigma_2 = \sigma_2(\gamma_2, K)$$

If σ_{1M} denotes the maximum of σ_1, σ_{2M} the maximum of σ_2 satisfying $Y_2 \leq 1$; C_1, C_2 are the fractions of $C(=1)$ assigned to the corresponding channels; then the maximum throughput for the configuration is:

$$\sigma_M = \begin{cases} C_1 \ (1+\alpha) \ \sigma_{1M} & , \quad \alpha \leq \dfrac{\sigma_{2M} \ C_2}{\sigma_{1M} \ C_1} \\ C_2 \ (1+\dfrac{1}{\alpha}) \ \sigma_{2M} & , \quad \alpha > \dfrac{\sigma_{2M} \ C_2}{\sigma_{1M} \ C_1} \end{cases} \qquad (12)$$

From (12) one can obtain that for a given α the channel split which maximizes the throughput, C_{1M} and C_{2M}, satisfy:

$$\frac{C_{2M}}{C_{1M}} = \frac{\alpha \sigma_{1M}}{\sigma_{2M}} \qquad (13)$$

or for the case studied here:

$$\frac{C_{2M}}{C_{1M}} = \frac{\alpha}{e(1-\frac{1}{K})^{K-1}} \qquad (14)$$

Figure 1 shows the maximum utilization vs. α for configurations 2-s-K and 1-s-K for K = 1, 3, and ∞. When the number of stations is increased, the maximum utilization decreases substantially, in particular for high values of α. This holds true for the 1-channel as well as the 2-channel configurations. In fact then K $\geq$ 3 then the maximum throughput is not much different from the case K = ∞.

Comparing a 2-channel and 1-channel configuration with the same number of stations, one can see that there is an interval of α for which the 2-channel configuration results in a higher maximum utilization. Outside of this interval the maximum utilization of the 1-channel configuration is higher. This interval of α decreases when the number of stations increases and constitutes a single point when K = ∞.

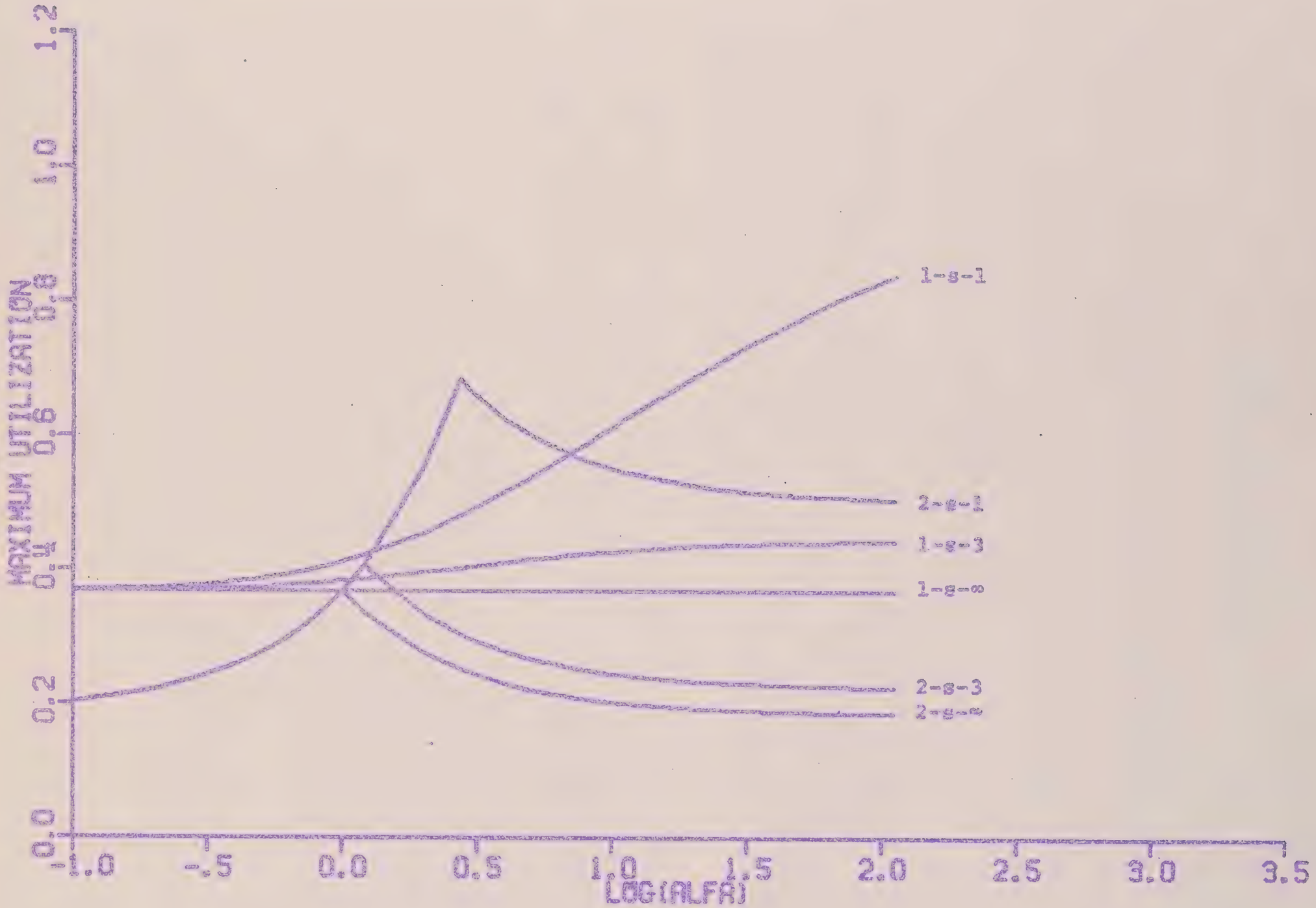
MAXIMUM UTILIZATION
0.0
0.2
0.4
0.6
0.8
1.0
1.2
-1.0
-.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
LOG(ALFA)
1-s-1
2-s-1
1-s-3
1-s-∞
2-s-3
2-s-∞

FIGURE 1

III. DELAY CONSIDERATIONS

The average delay from terminals to station D_1 and from station to terminal D_2 are given by:

$$D_1 = 1.5 + \bar{k}\,\frac{\gamma_1 - \sigma_1}{\sigma_1} \qquad (15)$$

$$D_2 = 1 + .5(1-\gamma_2) + \bar{k}\,\frac{\gamma_2 - \sigma_2/K}{\sigma_2/K} + \frac{\gamma_2}{2(1-\gamma_2)} \qquad (16)$$

Equations (15) and (16) hold true for the split channel as well as the common channel configurations. For configuration 2-s-1, $\gamma_2 = \sigma_2$ and the third term in equation (16), which represents the retransmission delay, vanishes. When $K \to \infty$, then $\gamma_2 \to 0$ in the second and fourth term in (16), and the third term becomes $\bar{k}(\gamma_2 - \sigma_2)/\sigma_2$ where now γ_2 is the rate for all stations.

The delay equations are drawn by considering γ_1 as the independent variable. That is, the channel traffic from terminals to stations, γ_1, is increased and it is required that the stations respond with packets for every packet that the stations correctly receive.

For several values of the parameter a we obtain the curves of average delay versus throughput σ. The delay is the weighted average of D_1 and D_2. The curves were drawn for $\bar{k} = 4$; that is, each time a packet is blocked it encounters an additional average delay of 4 packet transmission times. The delays in the curves are normalized to the number of packet transmission times in the common channel configuration, thus the configurations can be absolutely compared.

Figures 2, 3, and 4, show the average delay vs. the throughput per slot for the configurations considered, for $a = 0.5$, 2.5, and 10.0.

From the three values of α used, two values ($\alpha = 0.5$ and $\alpha = 10.0$) are in the region in which the 1-channel has a higher maximum throughput than the corresponding 2-channel. For $\alpha = 2.5$, configuration 2-s-1 has a higher throughput than 1-s-1. This can be verified from Figure 1. Note that in 2-channel configurations, either D_1, or D_2 go to infinity (unless at the value of α at which both directions "saturate" simultaneously). The form of the curves can be explained from Eqs. (15) and (16), and the requirement $\alpha = \sigma_2/\sigma_1$.

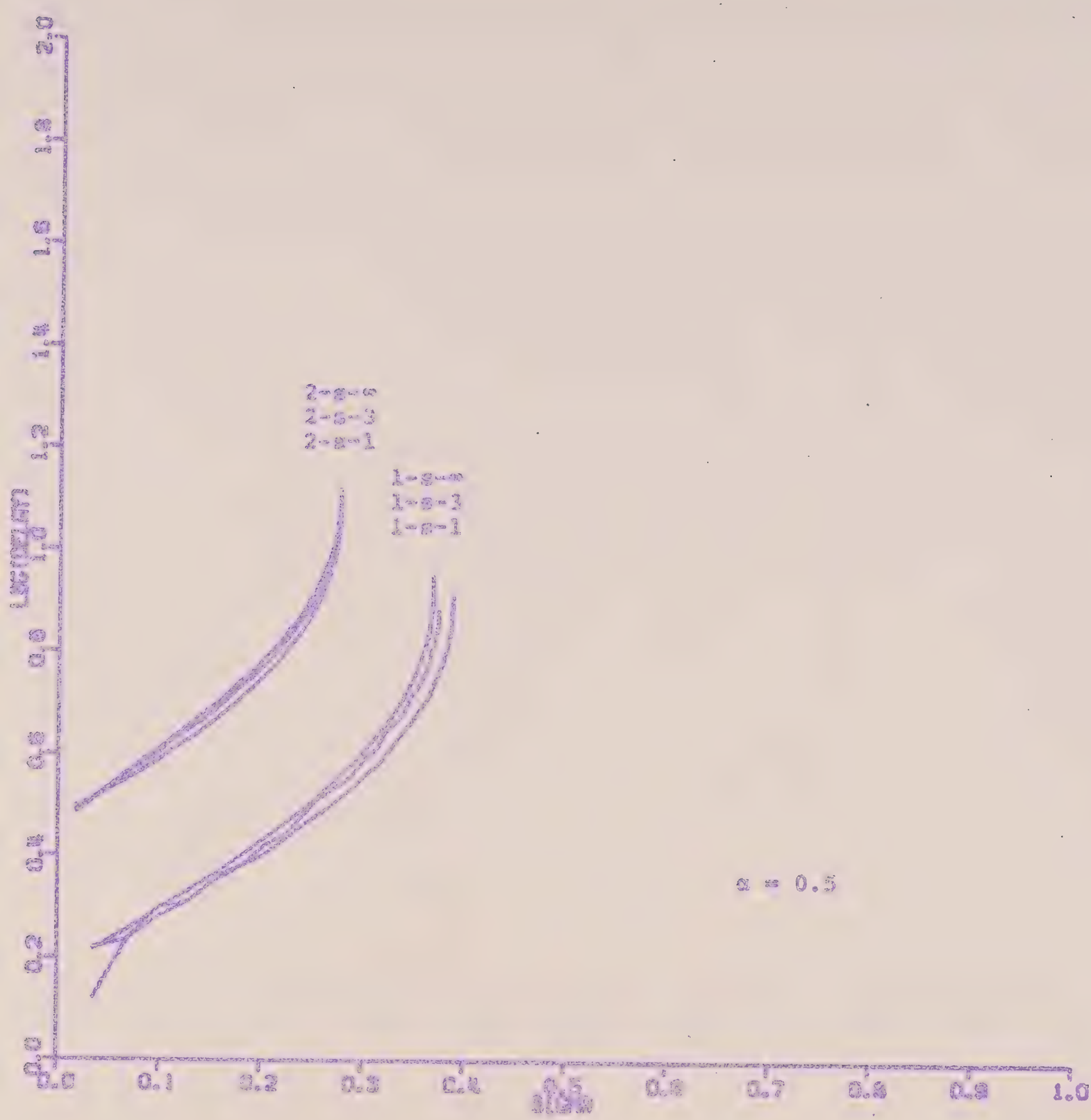
2-s-s
2-s-3
2-s-1
1-s-s
1-s-3
1-s-1
α = 0.5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

FIGURE 2

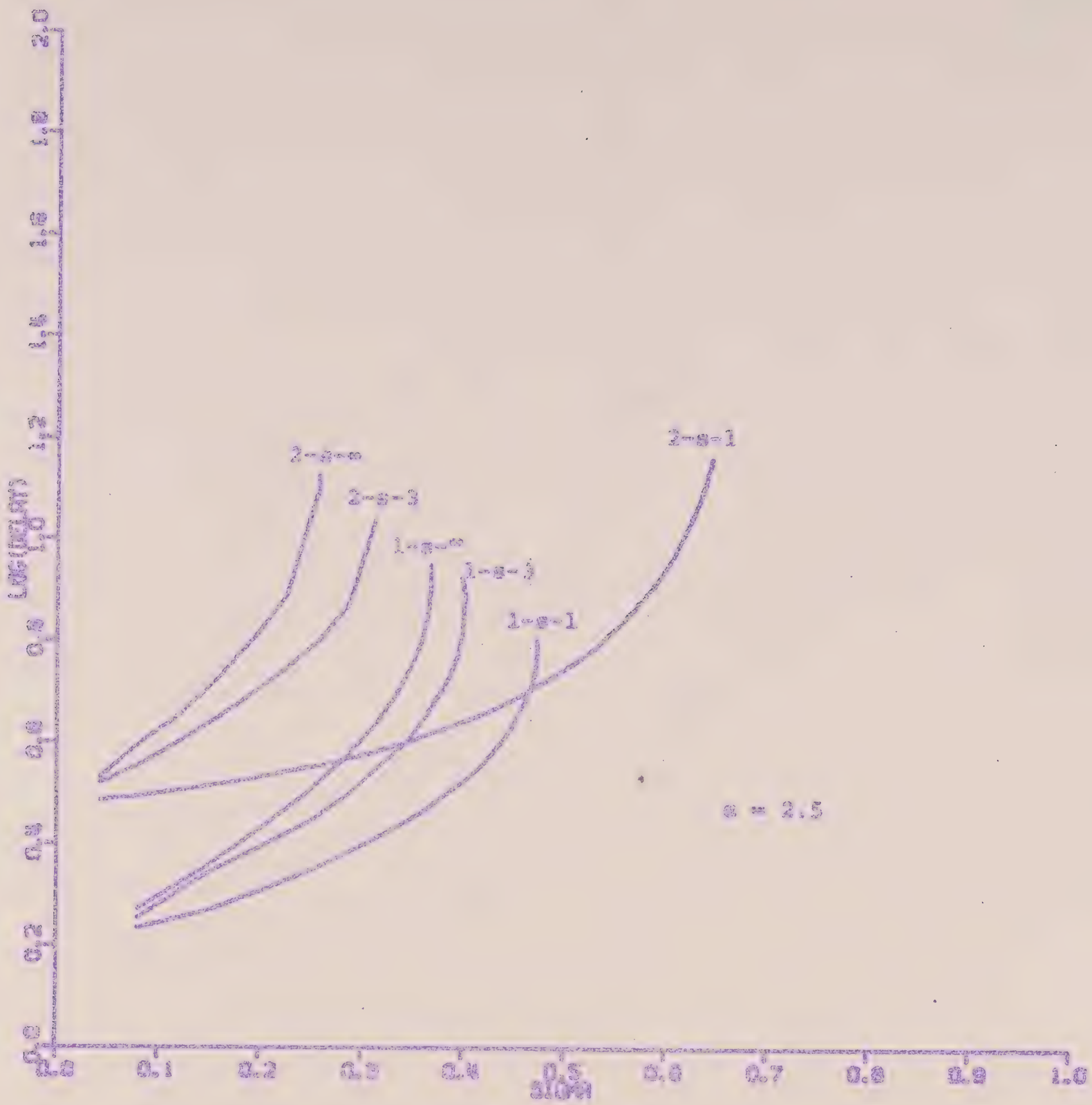

FIGURE 3

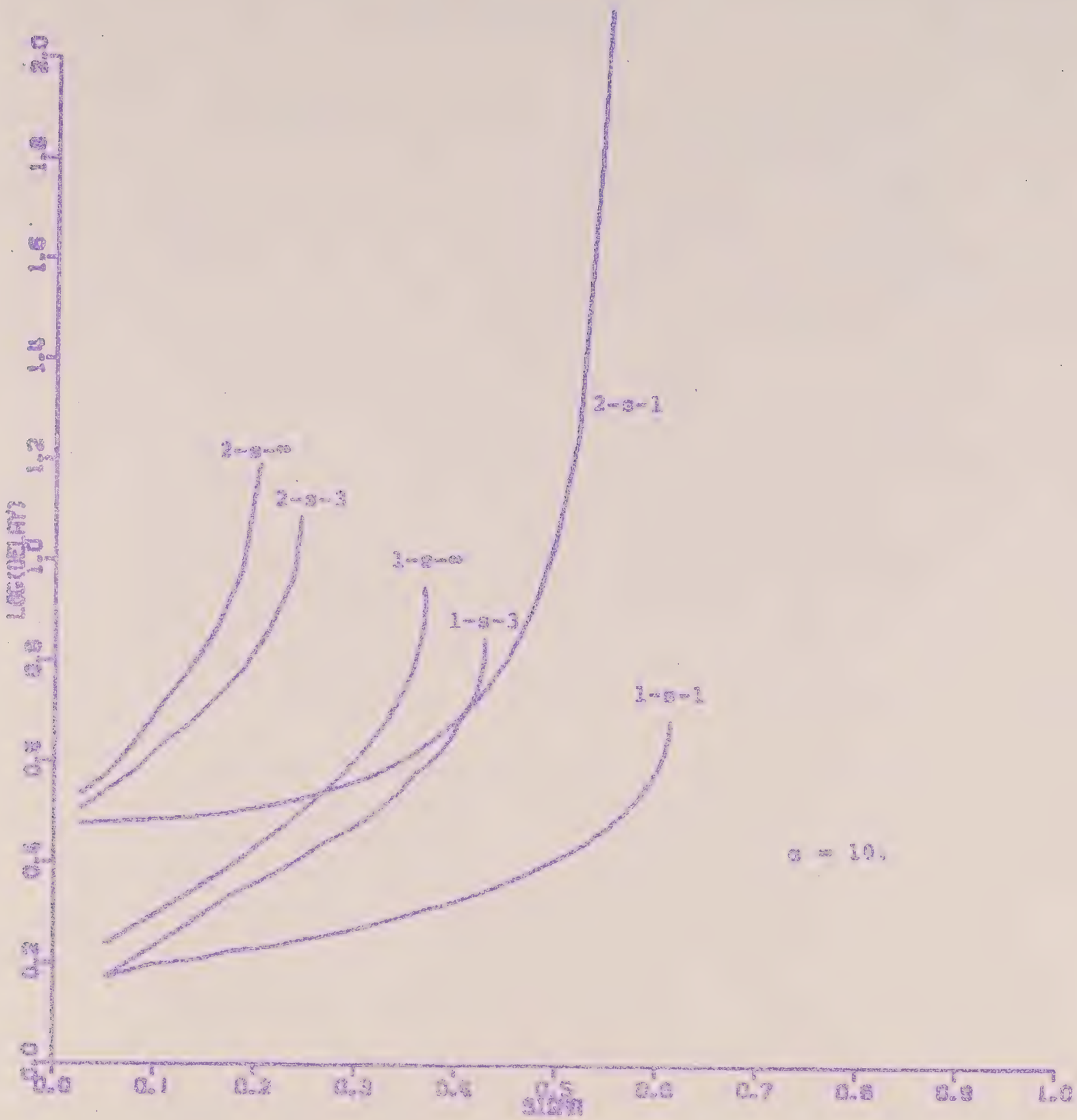
2-s-1
2-s-2
2-s-3
1-s-2
1-s-3
1-s-1
a = 10.
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0

FIGURE 4

IV. CONCLUSIONS

The conclusions related to the comparison between a 2-channel and a 1-channel configuration, with the same number of stations, is qualitatively the same as in [6]. Namely, there is an interval of α for which a higher maximum throughput can be obtained by the 2-channel configuration, however, when the system operates at lower throughputs then the 1-channel configuration results in lower values of delay. On the other hand, for α outside of this interval the 1-channel configuration is better in terms of maximum throughput and average delay. Furthermore, when the number of stations increases, then the interval of α discussed decreased, and the difference between the corresponding maximum throughputs in that same interval also decreases. Thus, the advantage that the 2-channel configuration has had diminishes for higher values of K. Consequently, the recommendation of a common channel configuration is further enhanced when the number of stations increases.

The maximum throughput decreases when the number of stations is increased for the 2-channel as well as the 1-channel configurations. The amount of reduction in throughput is quite substantial when the number of stations is increased from one to two or from two to three. Thus, it is recommended to use the smallest possible number of stations (apart from a similar recommendation when considering cost). This also implies that if several stations are to be located in an area then one should locate them so that the interference between packets transmitted from the stations is minimized.

V. REFERENCES

[1] Packet Radio-Systems Considerations; PRTN#5, Network Analysis Corporation, January 1, 1973.

[2] Packet Radio Broadcast Network-System Operation; PRTN #52 Network Analysis Corporation, June 1, 1973.

[3] N. Abramson: Excess Capacity of a Slotted ALOHA Channel; ASS Note #26, November 15, 1972.

[4] L. G. Roberts: Optimal Retransmission Delay for ALOHA System; ASS Note #9, July 1972.

[5] L. Kleinrock and S. S. Lam: Analytic Results with the Addition of One Large User; ASS Note #27, October 30, 1973.

[6] Channel Configuration for Packet Radio System; PRTN #45, Network Analysis Corporation, April 16, 1973.